

# The need to calculate detonation pressure profiles and use them as a source term in FE calculations to quantify the response of the containment subjected to detonations

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# Outline of talk

- Introduction
- For which  $p_{\text{det}}$ -scenarios seems the corresponding mechanical load  $p_{\text{stat}}$  to be clear
- For which  $p_{\text{det}}$ -scenarios is the corresponding mechanical load  $p_{\text{stat}}$  not yet clear
- What do experiments suggest
- Why are systematic experiments difficult
- Simple analytic approach („breathing mode“) to quantify radial displacement of wall in dependence of duration of detonative pressure pulse
- What is our hope concerning the mechanical load  $p_{\text{stat}}$  brought about by detonative pressure pulses
- What might help to answer the open questions with regard to mechanical load  $p_{\text{stat}}$

# Need to quantify the mechanical load associated with gas phase detonations

persistent efforts to increase productivity necessitate access to process parameters (P, T, gas phase composition) even in hitherto uncommon ranges of the explosion diagram

steadily increasing standards on process safety are to be heeded.

Process engineering has a vital interest to quantify the mechanical load associated with gas phase detonations as precise as possible

for all possible scenarios of  $p_{det}$  and for all possible geometries of the containment.

- stable detonation
- reflection of stable detonation
- DDT in „infinitely“ long pipes
- DDT short before reflection
- precompression
- „surface detonations“
- long pipes
- short pipes
- vessels
- vessel with attached pipes
- dry packings
- irrigated packings
- columns (alternating: packing - free space)
- bubble columns

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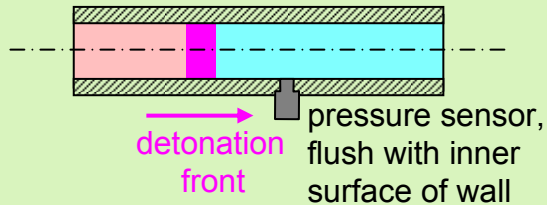
## Reminder: Definition of „static equivalent pressure“

- The „static equivalent pressure“ is the pressure  $p_{\text{stat}}$  applied in a hydraulic pressure test, which causes the same plastic deformation of the enclosure as a detonative pressure pulse, whose height is  $p_{\text{det}}$  (measured locally by e. g. piezoelectric pressure transducers or calculated)
- The *static equivalent pressure* is the basis for *explosion pressure resistant* or *explosion pressure shock resistant* design
- Experimental finding so far for steels with  $200 \text{ N/mm}^2 \leq R_{p0.2} \leq 250 \text{ N/mm}^2$ :

$$p_{\text{stat}} = \alpha * p_{\text{det}} \quad \text{with } \alpha < 1$$

# Scenario I: Side-on pressure of a stable Detonation in a pipe

(i. e. in plane perpendicular to shockfront)



$$p_{\text{det}} = p_{\text{initial}} * P_{\text{CJ}_r}$$

$P_{\text{CJ}_r}$  : denotes the *Chapman-Jouguet-pressure ratio* of the explosive gas mixture evaluated at the temperature the mixture had at the moment of ignition

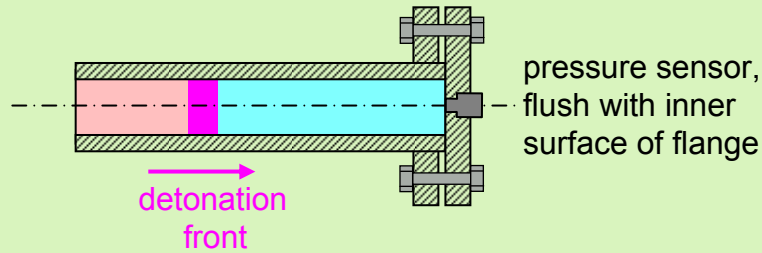
$p_{\text{initial}}$  : the pressure of the mixture at the moment of ignition

$$p_{\text{stat}} = \alpha_{\text{stab\_deto\_side-on}} * p_{\text{det}}$$

with  $\alpha_{\text{stab\_deto\_side-on}} = 0.6$

# Scenario II: Reflected pressure of a stable Detonation in a pipe

(i. e. in plane parallel to shockfront, e. g. at blind flange)



$$p_{\text{det}} = p_{\text{initial}} * P_{\text{CJ}_r} * F_{\text{reflec}}$$

$$2 \leq F_{\text{reflec}} \leq 2.5$$

$$p_{\text{stat}} = \alpha_{\text{stab\_deto\_reflected\_pressure}} * p_{\text{det}}$$

with  $\alpha_{\text{stab\_deto\_reflected\_pressure}} = 0.6$

## Important:

Furthermore it should be born in mind that the shock reflected at the blind flange propagates backwards into the taylor expansion fan of the detonation front that had arrived at the blind flange just before. Close to the blind flange the side-on pressure of the reflected shock is about twice the side on pressure of the original detonation. When the reflected shock has propagated backwards over a distance equal to the width of the taylor expansion fan, the side-on pressure it exerts on the wall has dropped to the value of the side-on pressure of the original detonation.

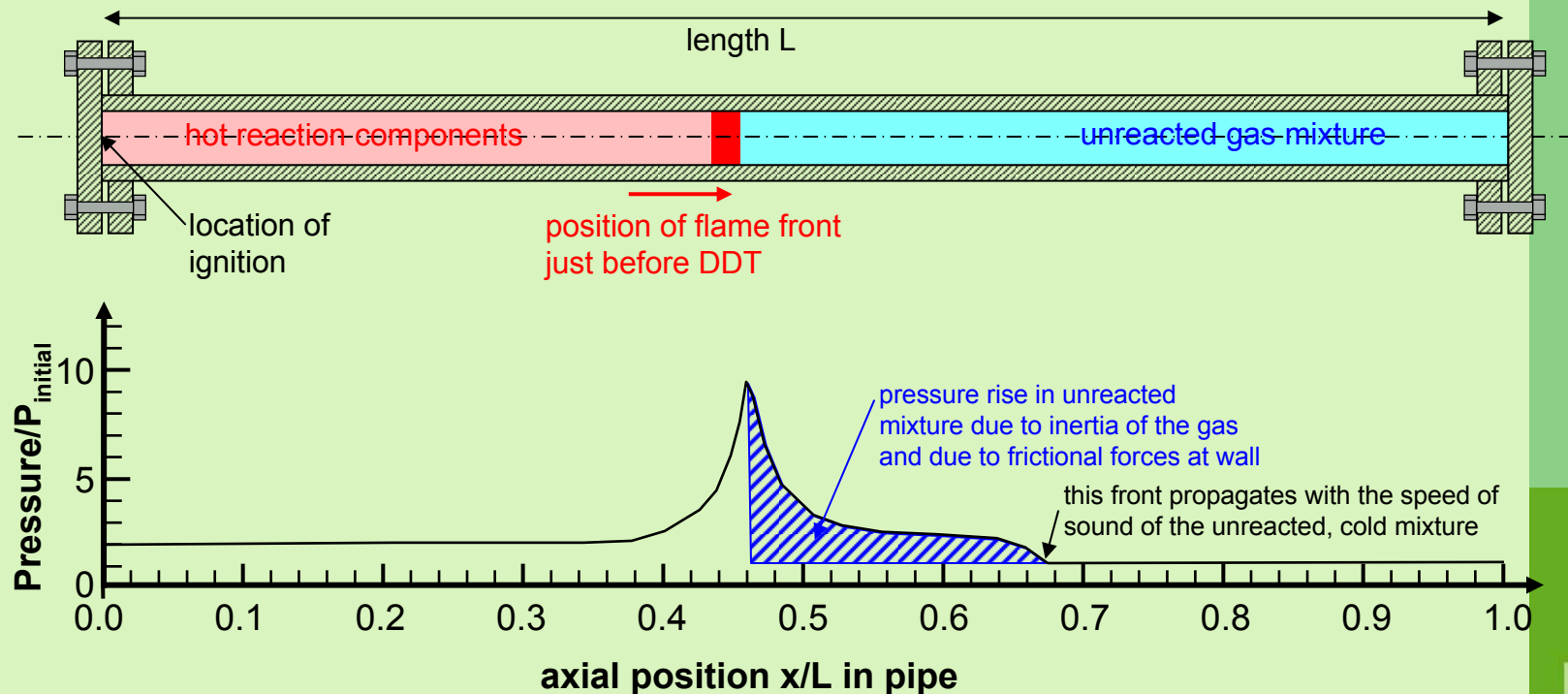
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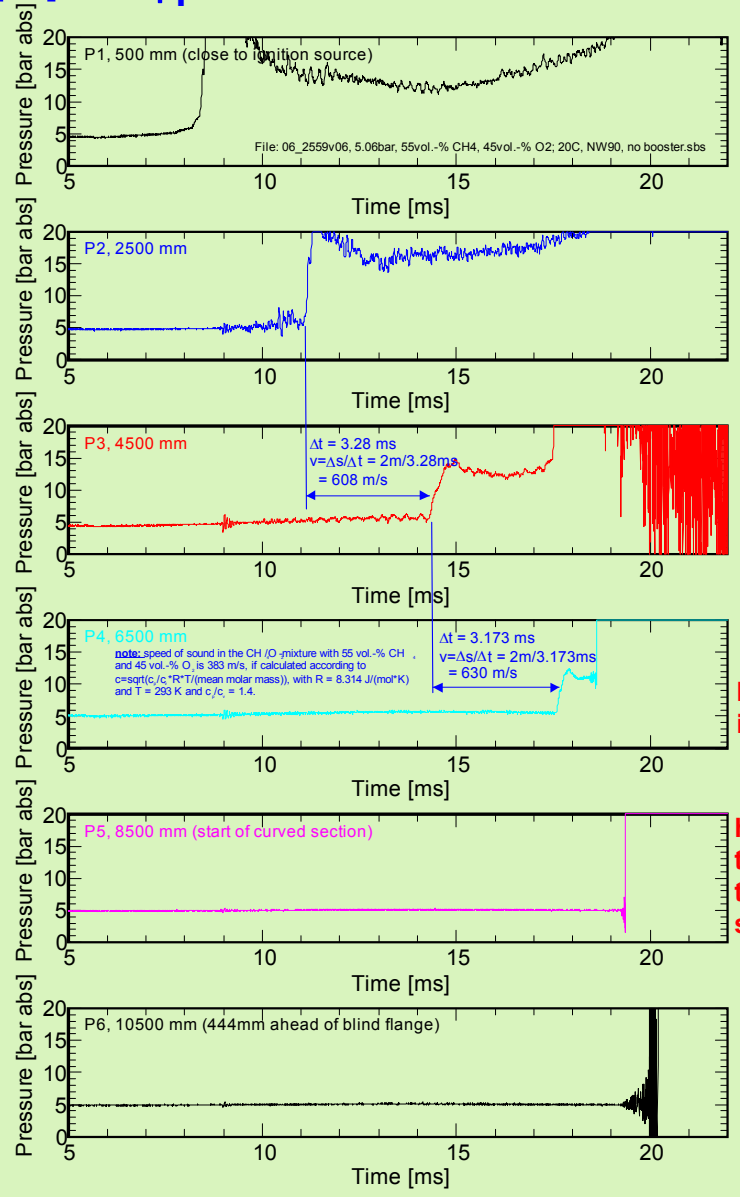
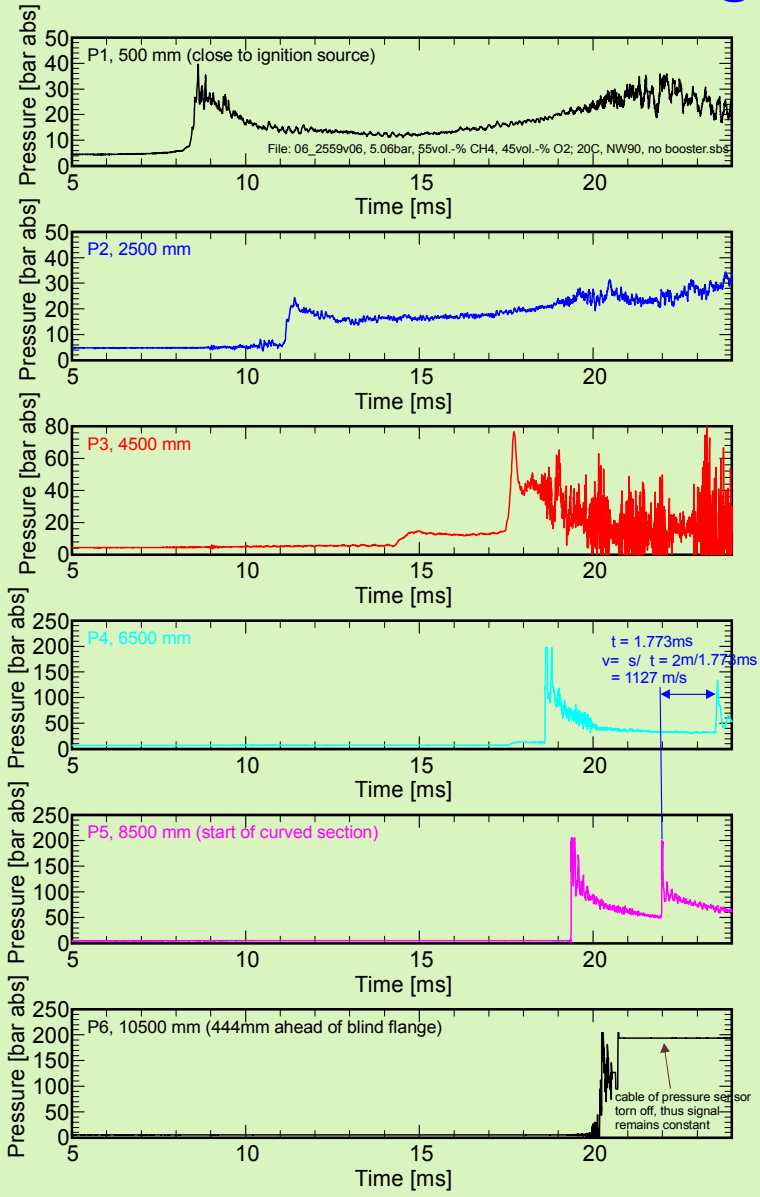
# Scenario III: Side-on pressure in a long pipe at the location where the DDT occurs

meaning of „long“: pressure waves send out during initial deflagrative stage of explosion have not yet reached the blinded end of the pipe at the moment when the DDT occurs and will even not arrive at the blinded end before the detonation does.

Qualitative sketch of this scenario (just before DDT happens):



# Example for scenario III: DDT in CH<sub>4</sub>:O<sub>2</sub>=55:45 mol:mol, 5 bar abs, 20°C, 11 m long pipe, $\phi_i = 86$ mm



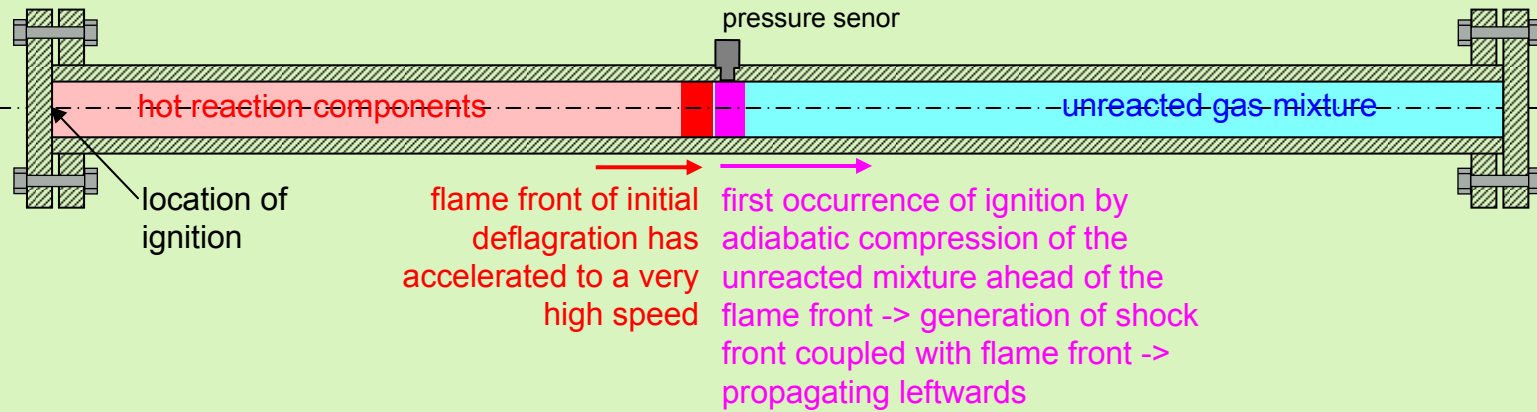
at 17.7 ms DDT is just before completion, here in a region where unburned gas, which has an inertia, was present at about twice it's initial pressure because it got accelerated by the expanding reaction gases

here the detonation is established

here detonation has overtaken the pressure wave caused by the very initial deflagrative stage of the reaction



# Scenario III: Side-on pressure in a long pipe at the location where the DDT occurs (continued)



$$p_{det} = p_{initial} * P_{CJ_r} * F_{DDT}$$

$F_{DDT} = ?$  (some people assume a factor of about 8)

$$p_{stat} = \alpha_{DDT\_long\_pipe} * p_{det}$$

$\alpha_{DDT\_long\_pipe} = ?$

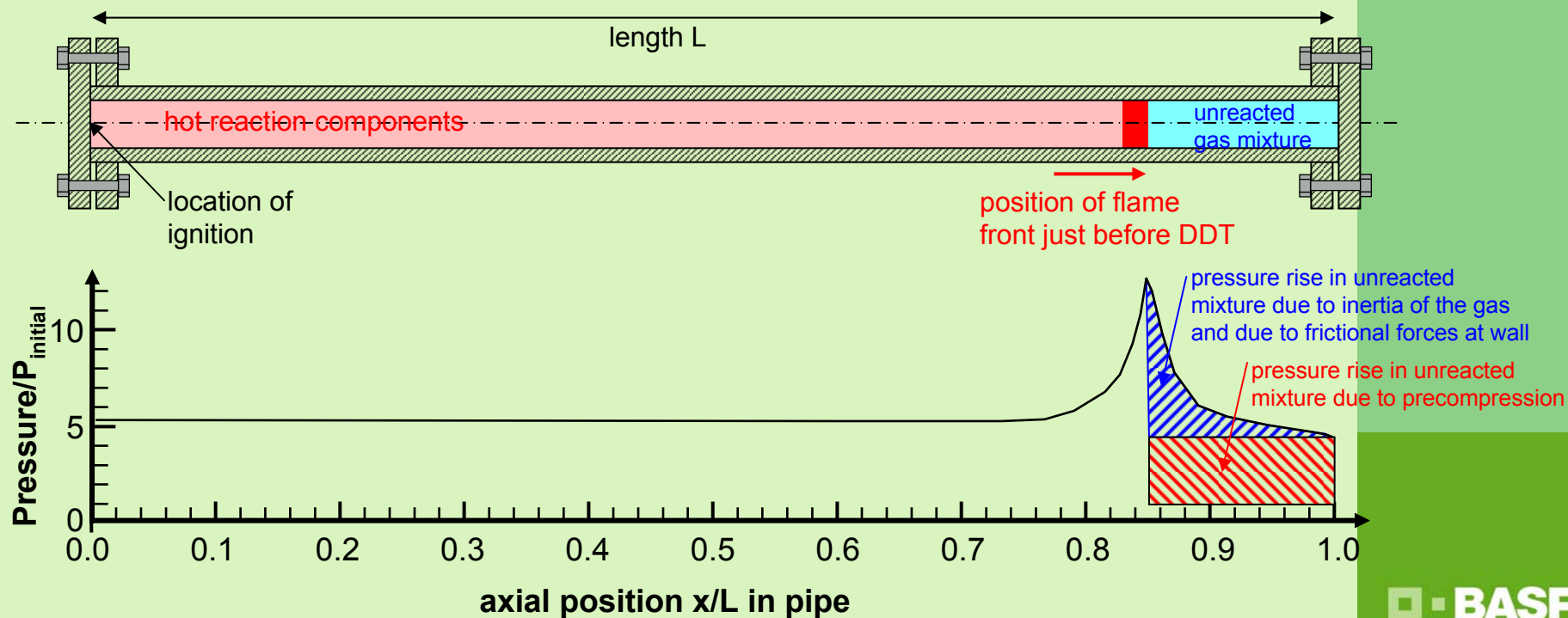
Limited number of experiments suggest that  $p_{stat}$  at the DDT in a long pipe is not larger than 1.75 times the value of  $p_{stat}$  for the stable detonation.  
 If this were generally true and if  $F_{DDT}$  were really 8, then  $\alpha_{DDT\_long\_pipe}$  would be  $1.75 * 0.6/8 = 0.13$

# Scenario IV: Side-on pressure in a pipe at location where DDT occurs in case that the pressure waves sent out during initial deflagrative stage of the explosion have already reached the blinded end

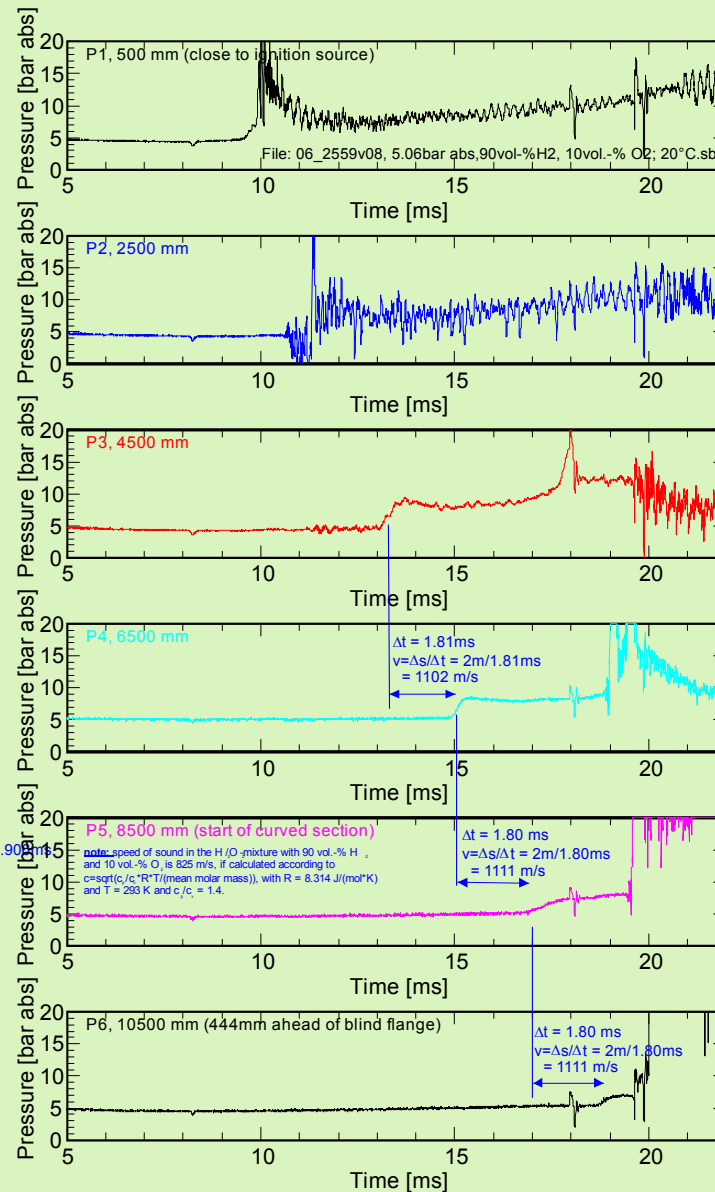
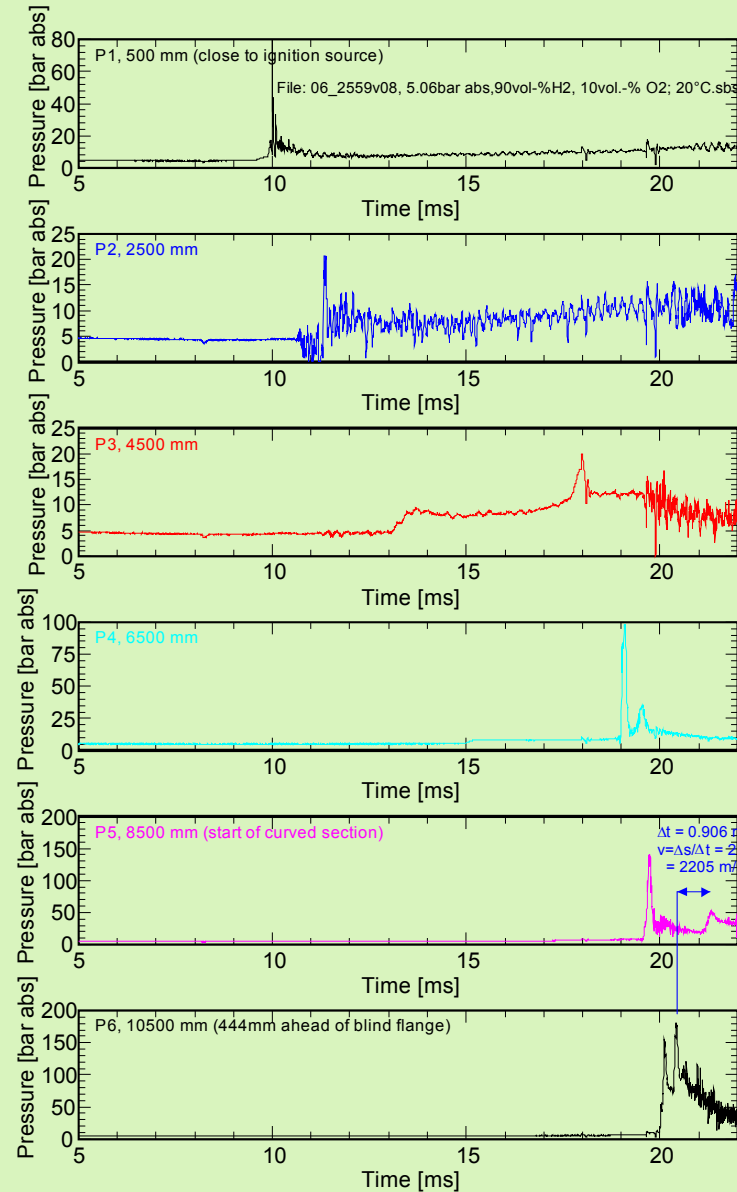
## Note:

This scenario is usually referred to as „DDT in precompressed mixture“. The contribution to the pressure rise in the unreacted mixture due to precompression can vary between 0 and  $P_{initial} *$  (deflagration pressure ratio).

Qualitative sketch of this scenario (just before DDT happens):



# Example for scenario IV in a pipe: DDT in H<sub>2</sub>:O<sub>2</sub>=90:10 mol:mol, 5 bar abs, 20 °C, 11 m long pipe, $\phi_i = 86$ mm (1/2)

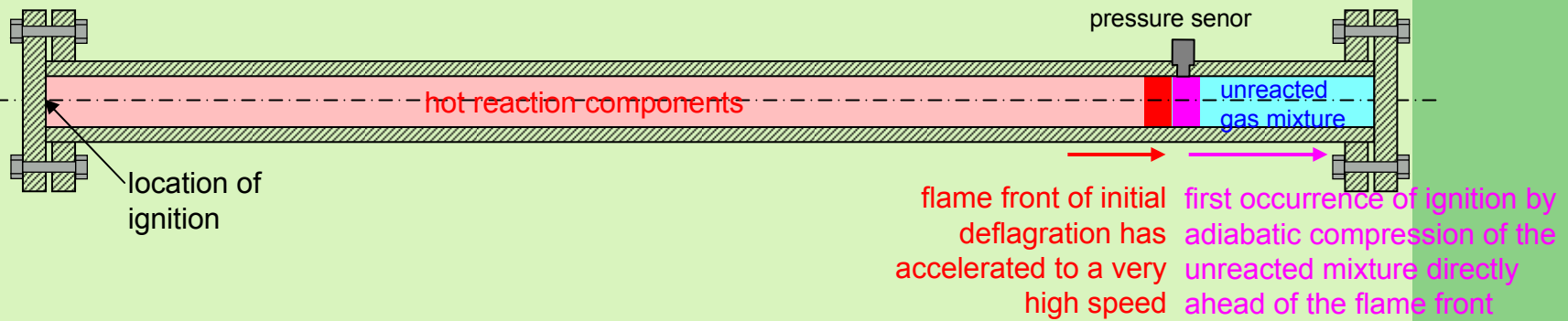


at 18.0 ms DDT starts to emerge in a region where unburned gas, which has an inertia, was present at about twice it's initial pressure because it got accelerated by the expanding reaction gases

DDT has occurred

here detonation has not yet overtaken the pressure wave caused by the very initial deflagrative stage of the reaction. Presumably it will not overtake it in the last 500 mm of the pipe.

# Scenario IV: Side-on pressure in a pipe at location where DDT occurs in case that the pressure waves sent out during initial deflagrative stage of explosion have already reached the blinded end. (continued)



## Tentative equations:

$$p_{\text{det}} = p_{\text{initial}} * P_{\text{CJ}_r} * F_{\text{DDT}} * F_{\text{precomp}} * F_{\text{temp}}$$

with  $F_{\text{precomp}}$  and  $F_{\text{temp}}$  as defined in talk at 41st UKELG-meeting

$$p_{\text{stat}} = \alpha_{\text{DDT\_short\_pipe}} * p_{\text{det}}$$

$$\alpha_{\text{DDT\_short\_pipe}} = ?$$

## Open questions and problems:

- If the pressure front that propagates with the speed of sound arrives at the blind flange, is reflected and propagates backwards, how will it influence the DDT?
- Can one use  $F_{\text{precomp}}$  and  $F_{\text{temp}}$  as defined in talk given at 41st UKELG-meeting?
- Can we use the same value of  $F_{\text{DDT}}$  as in scenario III or will  $F_{\text{DDT}}$  depend on  $F_{\text{precomp}}$  and  $F_{\text{temp}}$ ?
- What will be the final relation between  $p_{\text{stat}}$  and  $p_{\text{det}}$ , i. e. what will be  $\alpha_{\text{DDT\_short\_pipe}}$ ?
- If for a given geometry and a given mixture it can not be excluded that scenario IV will happen instead of scenario III and since the point where the DDT occurs can never be predicted exactly, one might always have to assume the maximum value for  $F_{\text{precomp}}$ , i. e. the deflagration pressure ratio.

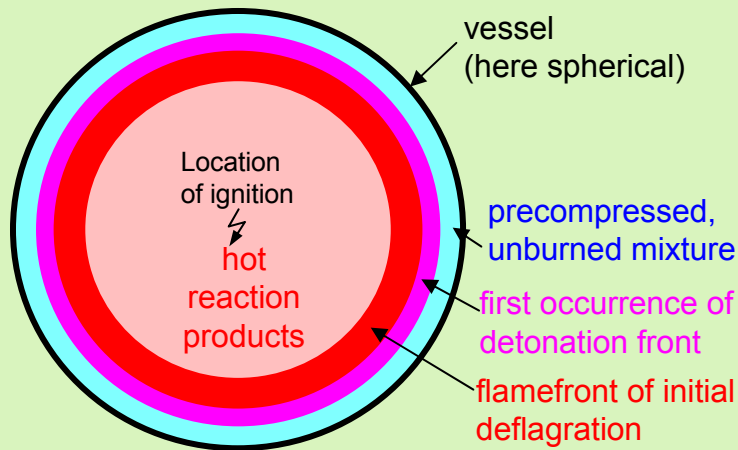
# Comment to scenario IV

- Scenario IV will be rare in pipes, because they are usually much longer than the predetonation distance.  
This means that in most cases scenario III occurs and not scenario IV.
- However, if a DDT occurs in an empty vessel, scenario IV will occur in most cases instead of scenario III, because typical vessel diameters are of the order of the predetonation distance.

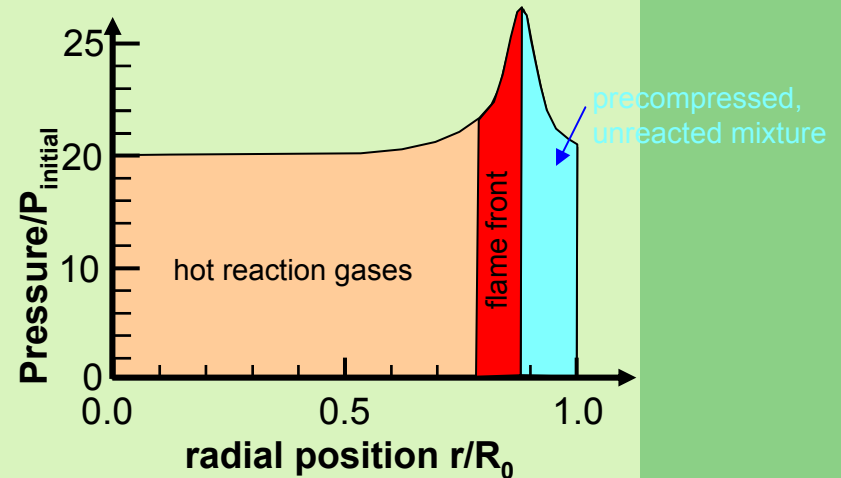
# Example for scenario IV : DDT in a 20 l sphere

**Gas mixture:** Propene = 38 vol.-%, O<sub>2</sub> = 62 vol.-%, P<sub>initial</sub> = 5 bar abs, T<sub>initial</sub> = 25°C, ignition in center, deflagration pressure ratio = 22 (see talk given at 41st UKELG-meeting)

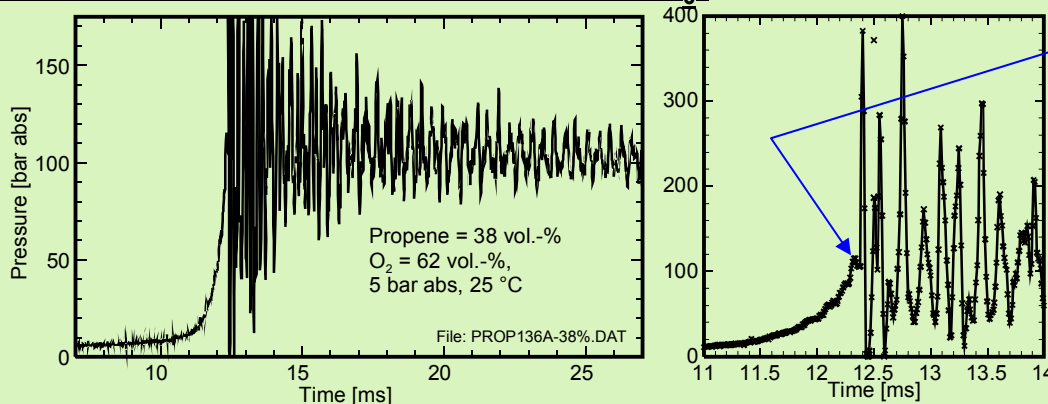
## Qualitative sketch of spacial distribution of gases inside sphere at the moment of DDT



## Qualitative sketch of radial pressure distribution directly before DDT occurs



## Pressure measured in wall (i. e. at r = R<sub>0</sub>)







# Some explanations to the calculations

## Sketch for understanding what is meant with V1, V1-expanded, V2 and V2-compressed

1. step: V1 and V2 contain the same explosive mixture. There is no membrane between V1 and V2. In a Gedankenexperiment we let only V1 react, but do not yet allow expansion of the reaction gases. V1 now contains the hot reaction gases at high pressure, V2 still contains the unreacted mixture at the initial pressure and temperature



2. step: Now we allow V1 to expand at the expense of V2 until pressure equilibrium is attained, which means that the pressure in the expanded volume V1 (labelled "V1-expanded") is equal to the pressure in the adiabatically compressed volume V2 (labelled "V2-compressed")

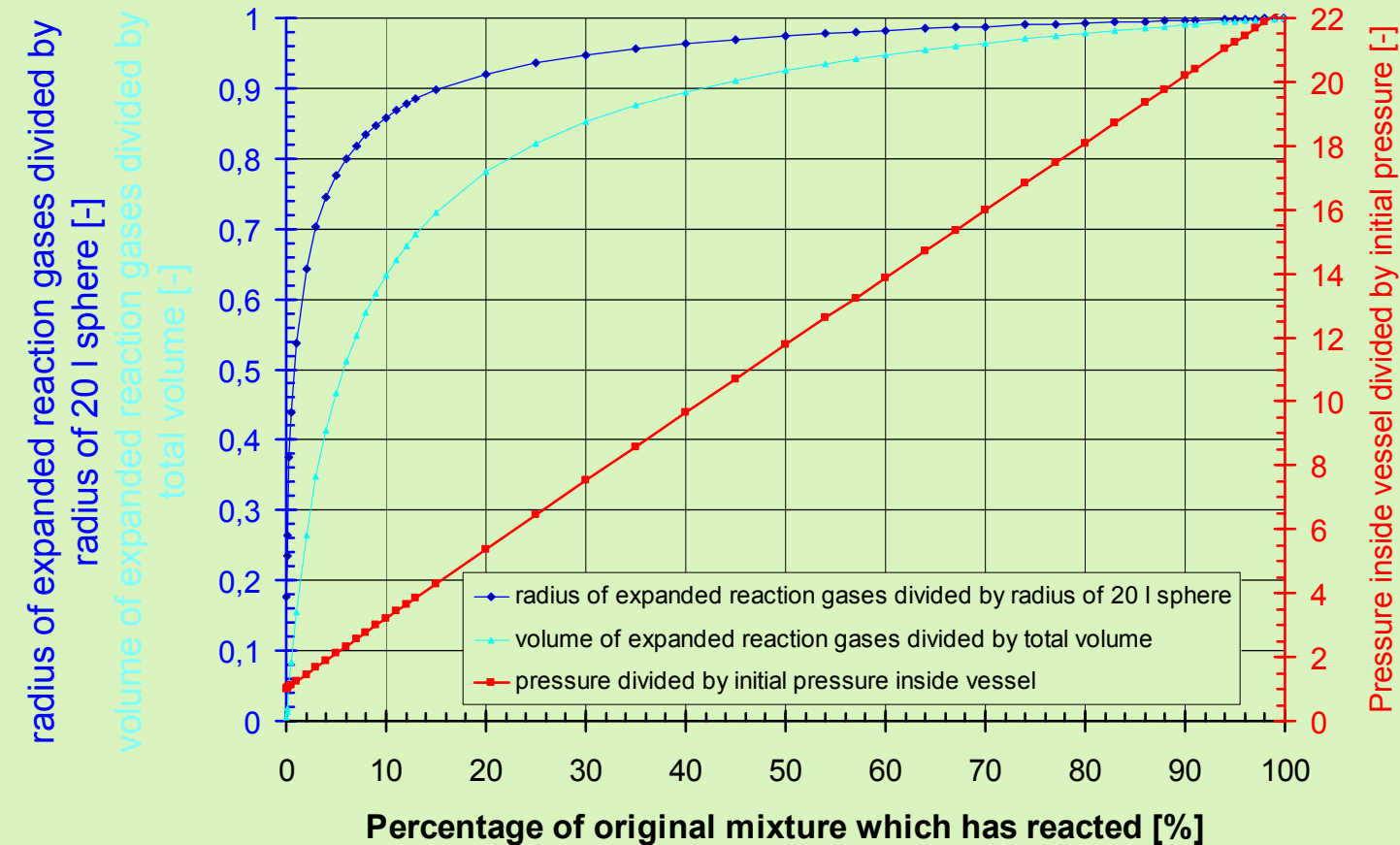


(temperature of reaction gases has dropped slightly due to expansion, i.e. the work that had to be brought up for compressing V2)

(temperature of the yet unreacted mixture in V2-compressed has risen slightly due to the adiabatic compression brought about by the expansion of the reaction gases in Volume V1-expanded)

# Further information to DDT in 20 l sphere: Rise in volume of the expanding reaction gases and rise of pressure inside sphere during the deflagrative explosion inside the sphere

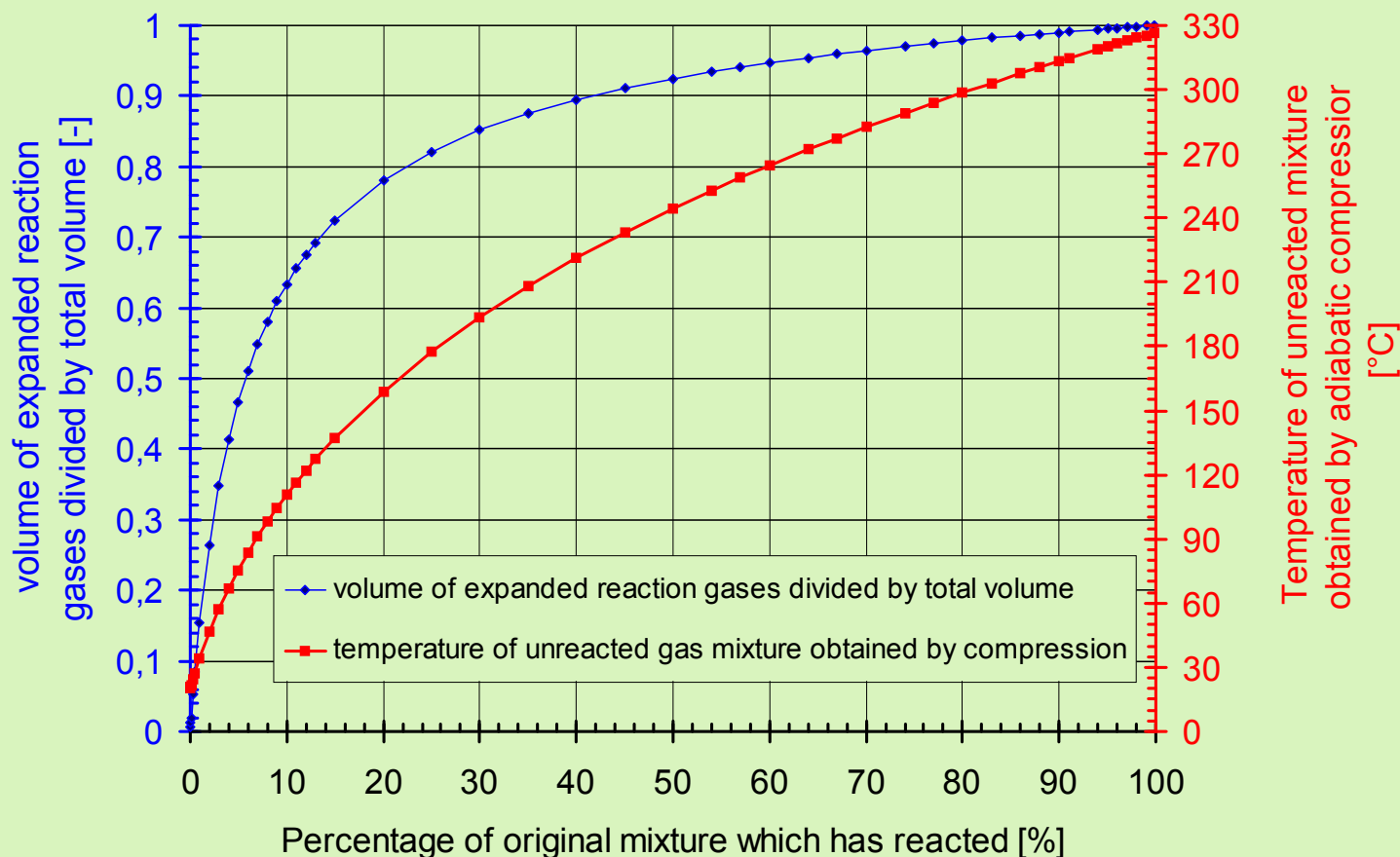
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**Note.** When precompression has reached a factor of 20, the shell of unreacted precompressed mixture is extremely thin in radial direction. The DDT probably does not happen over an entire spherical surface at the same instant, but only over a sub-surface. So there is a good chance that the first pressure peak „seen“ by the pressure sensor is not the detonation peak but the reflected shock wave coming from the detonation that hit the wall elsewhere.

# Further information to DDT in 20 l sphere: Rise in volume of expanded reaction gases and rise of temperature in the unreacted mixture during the deflagrative explosion inside the sphere

Volume of expanded reaction gases divided by total volume of vessel and temperature in unreacted mixture obtained by adiabatic compression (Data for Propene:O<sub>2</sub> = 38 vol.-% : 62 vol.-%, P<sub>initial</sub> = 5 bar abs and T<sub>initial</sub> = 20°C in 20 l sphere,  $\gamma = 1.3$ )



# Further scenarios which are unclear

- Scenario IV with reflection:

If the detonation established in scenario IV hits the blind flange and gets reflected, what will be  $p_{\text{stat}}$  at the blind flange?

(since  $p_{\text{stat}}$  will presumably decrease when the detonation propagates from the point, where it came into being, to the blind flange,  $p_{\text{stat}}$  at the blind flange will most likely depend on the distance between the DDT point and the blind flange. However, because the location of the DDT is not exactly predictable, one would have to assume the worst-case scenario, which is the DDT directly ahead of the blind flange. This means maximum precompression).

- detonation in columns (alternating: packing - free space)

- detonation in bubble columns

- „surface detonations“

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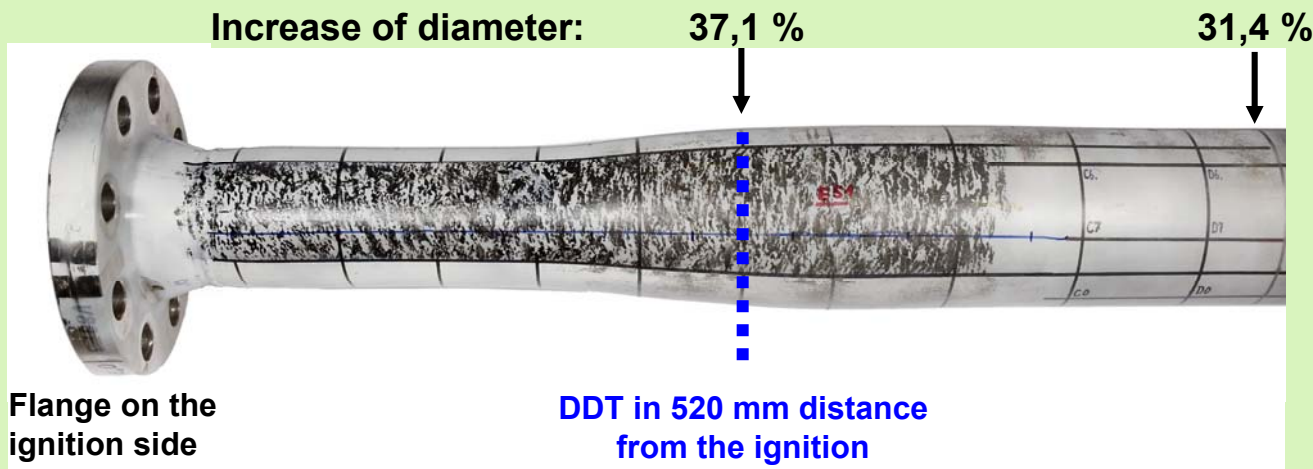
# Example 1: DDT and stable detonation (W. Stadtmüller)

## Parameters of 114.3 x 6.02 pipe made out of 1.4541:

- wall thickness  $s = 6.02$  mm
- inner diameter  $\phi_i = 102.26$  mm
- yield strength  $R_{p0.2} = 230$  N/mm<sup>2</sup> at 25°C (sample cut out of the wall)
- ultimate tensile strength  $R_m = 590$  N/mm<sup>2</sup> at 25°C (sample cut out of the wall)
- pressure generating a hoop stress equal to  $R_{p0.2}$ :  $P_{Rp0.2} = 2*s*R_{p0.2}/\phi_i = 270.8$  barg
- burst pressure in a hydraulic test: 519 bar

## Conditions for detonation test:

80 vol.-% H<sub>2</sub>:O<sub>2</sub>=2:1, 20 vol.-% N<sub>2</sub>, initial pressure 70 bar, 20°C,  $P_{CJ} = 1470$  bar,  $P_{CJ}/P_{initial} = 21$

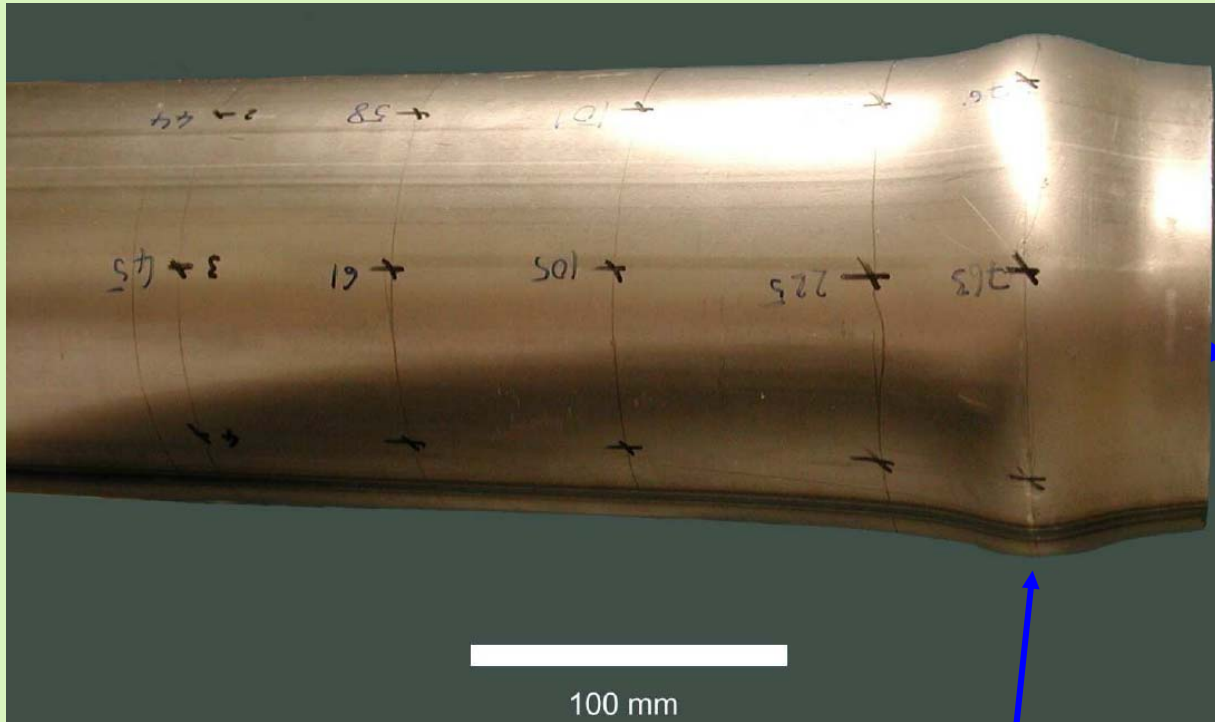


Since detonative pressure at DDT is much larger than  $P_{CJ}$ , the ratio between the static equivalent pressure  $p_{stat}$  and the detonative pressure  $p_{det}$  at the DDT is much less than  $519/1470 = 0.353$

## Reference:

W. Stadtmüller, E. Roos, S. Offermanns, Materialprüfungsanstalt University Stuttgart, private communication within the scope of the reactor safety research of the German Federal Ministry of Economics and Technology (BMW). Parts of the research work will be published at the *International Conference on Structural Mechanics in Reactor Technology SMIRT 20 – Conference*, August 09-14, 2009, Helsinki, Finland

## Example 2: DDT short before blind flange (J.E. Shepherd)



The peak pressure measured at the end was approximately 500 bar according to Shepherd

15% plastic deformation

### Tube parameters:

- wall thickness  $s = 1.5$  mm
- inner diameter  $\phi_i = 129$  mm
- yield strength  $R_{p0.2} = 305$  N/mm<sup>2</sup> (AISI1010 steel)
- pressure generating a hoop stress equal to  $R_{p0.2}$ :  $P_{Rp0.2} = 2*s*R_{p0.2}/\phi_i = 70.9$  barg

Since static pressure yielding 15% deformation is of the order of  $1.5*P_{Rp0.2}$ , the ratio between the static equivalent pressure  $p_{stat}$  and the detonative pressure  $p_{det}$  at the DDT is about  $(1.5*70.9)/500 = 0.21$

### Reference:

J.E. Shepherd, "STRUCTURAL RESPONSE OF PIPING TO INTERNAL GAS DETONATION", Proceedings of PVP2006-ICPVT-11, 2006 ASME Pressure Vessels and Piping Division Conference, July 23-27, 2006, Vancouver BC, CANADA

H.-P. Schildberg, 43rd UKELG meeting, 24th June 2009, Imperial College, London



# Example 3: DDT directly ahead of blind flange (BASF) (1/2)

picture of reactor used for the tests:



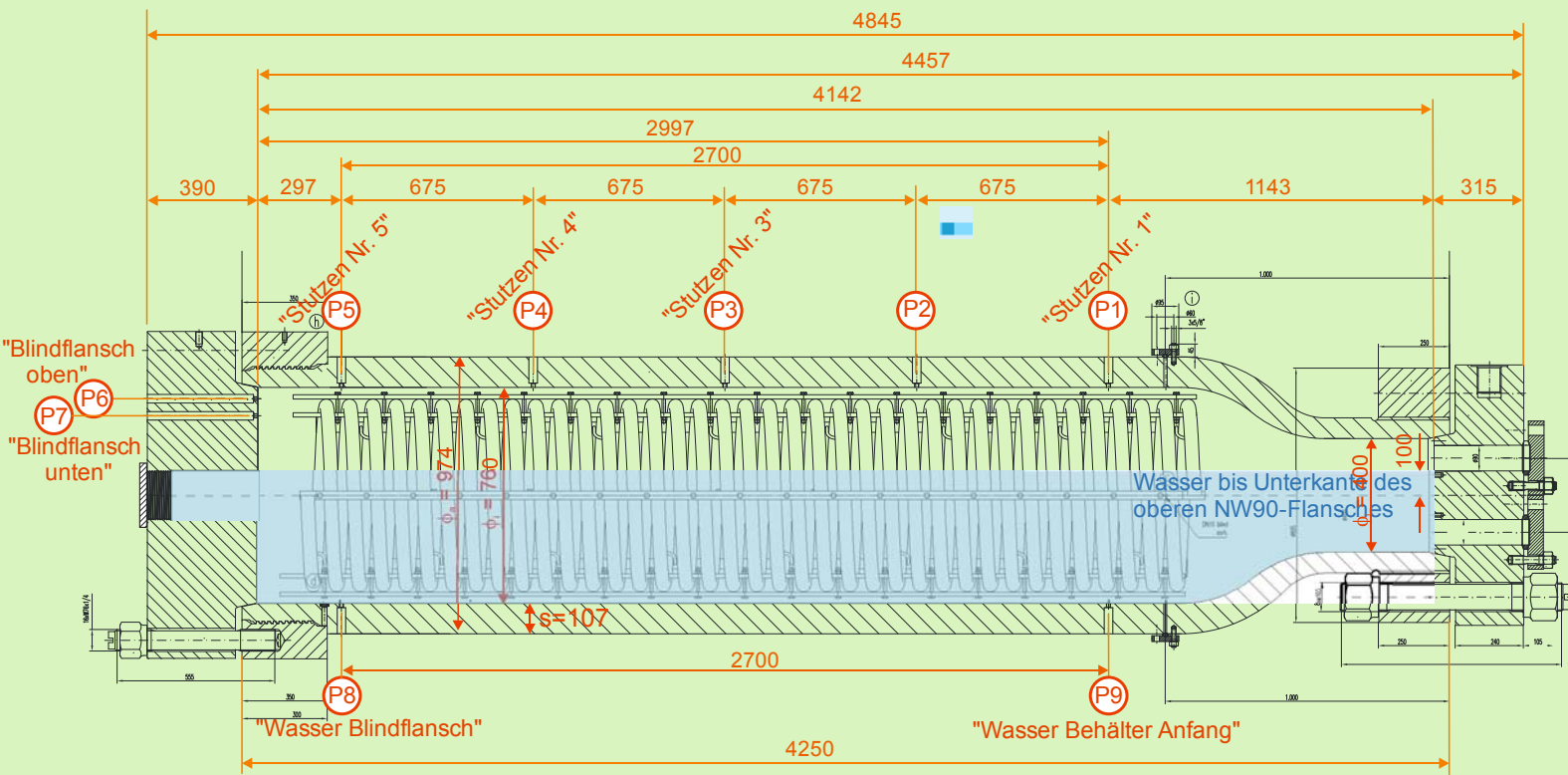
# Example 3: DDT directly ahead of blind flange (BASf) (2/2)

**Parameters of 974 x 107 pipe reactor:**

- wall thickness  $s = 107 \text{ mm}$
- inner diameter  $\phi_i = 760 \text{ mm}$
- wall material Ce36,  $R_{p0.2} = 506 \text{ N/mm}^2$ ,  $R_m = 691 \text{ N/mm}^2$  at  $25^\circ\text{C}$
- pressure generating a hoop stress equal to  $R_{p0.2}$ :  $P_{Rp0.2} = 2 \cdot s \cdot R_{p0.2} / \phi_i = 1424 \text{ barg}$

**Conditions for detonation test:**

15 bar  $\text{C}_2\text{H}_2$ ,  $20^\circ\text{C}$ ,  $P_{CJ}/P_{\text{initial}} = 20$  at  $20^\circ\text{C}$ , ignition at position of pressure sensor P3, DDT occurred between P5 and left blind flange, mixture was precompressed to at least 110 bar, pressure recording of P6 und P7: 5300 bar



**Oberservation: no deformation of blind flange or wall, not even lift-off of blind flange**



# Summary

Although at the DDT the value of  $p_{\text{det}}$  is presumably much larger than the value of  $p_{\text{det}}$  of the stable detonation, the static equivalent pressure at the DDT is presumably not by the same factor larger than the static equivalent pressure of the stable detonation.

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# Difficulties with experiments for clarifying the load in scenarios III and IV

- Information about  $p_{\text{stat}}$  can only be obtained by conducting experiments at such high initial pressures that permanent deformations are generated in the pipe wall. Since repeated detonative loads at the limit of or even beyond the elastic range of the wall material changes its mechanical parameters (strain-hardening („Kaltverfestigung“) and embrittlement („Versprödung“); see example with 100 l vessel), one would have to replace the test pipes at best after each detonation => very expensive and very time consuming.
- When trying to realize scenario IV in a test, the number of tests will be extremely large, because most tests will end up with scenario III or will not show a DDT at all.
- Even if scenario IV does occur in some experiment, it is not as well defined in the mechanical load as scenario III, because the degree of precompression can take any value between 0 and the deflagration pressure ratio.

# Example for embrittlement of the wall of vessels by repeated exposure to strong detonative loads

new vessel

## Vessel parameter:

V = 100 l, design pressure: 64 bar,  $\phi_o = 355.6$ ,  $\phi_i = 327.2$ ,  $s=14.2$ ; Material of cylindrical section: St35.8 = 1.0305,  $R_{p0.2}=339 \text{ N/mm}^2$ ,  $R_m = 453 \text{ N/mm}^2$  (tested was 1 pipe of the charge of  $355.6 \times 14.2$  pipes),  $P_{Rp0.2} = 294 \text{ bar}$

## Conditions for the detonation tests:

- About 20 tests with stoichiometric Ethylene/O<sub>2</sub> at initial pressures of 2 to 8 bar abs. Then vessel was filled with Raschig-Rings 15 mm x 15 mm,  $s = 0.3 \text{ mm}$
- Test no. 19:  $P_{\text{initial}} = 18 \text{ bar abs} \Rightarrow 2 \%$  diameter increase;
- Test no. 20:  $P_{\text{initial}} = 20 \text{ bar abs} \Rightarrow$  rupture (see picture below).

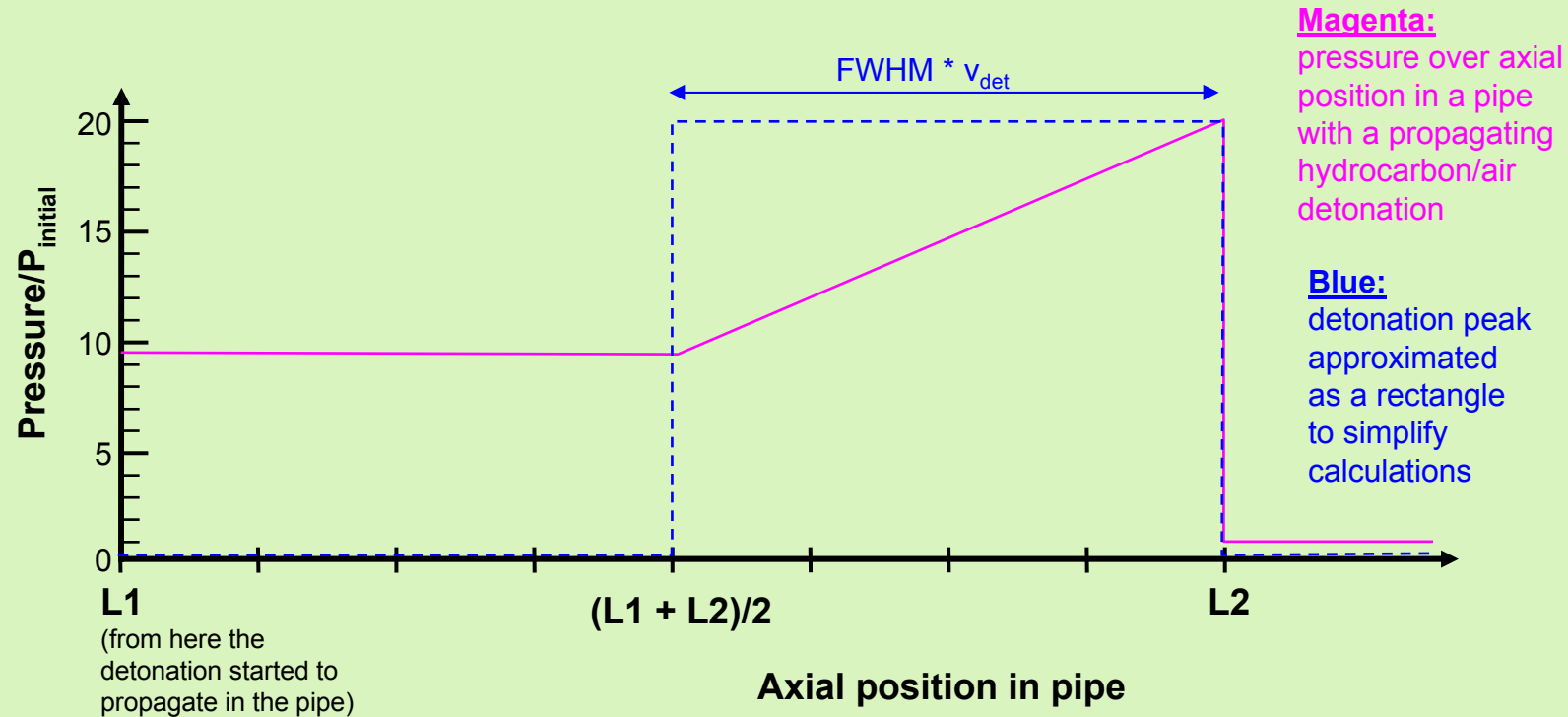
An increase of 10% in  $P_{\text{initial}}$  should not result in a transition from 2% diameter increase to vessel rupture unless the material had become brittle by all the preceding tests at rather high pressure loads.



vessel after  
test no. 20

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# Approximation of detonative pressure pulse by rectangular peak of duration FWHM



**Note:**  $\Delta s$ : distance the detonation has travelled

$v_{\text{det}}$ : detonation speed

$\phi_i$ : inner pipe diameter

Approximately the following holds as long as  $\Delta s < 70 * \phi_i$ :  $\text{FWHM} \cong 0.5 * \Delta s / v_{\text{det}}$

For  $\Delta s > 70 * \phi_i$  FWHM will usually no longer increase, i.e. constant pressure/space profile



# Definition of parameters for calculating the pipe response to a detonative load with a simplified pressure/time profile

## Parameters defining the pipe:

$R_i$  = inner radius

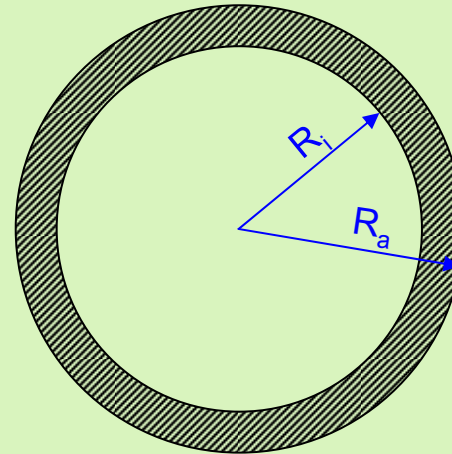
$R_o$  = outer radius

$s$  = wall thickness =  $R_o - R_i$

$\rho$  = density of pipe material, typical  $7800 \text{ kg/m}^3$

$E$  = E-modulus of pipe material, typical  $200000 \text{ N/mm}^2$

$r(t)$  = displacement of the wall in radial direction from the position at rest, i.e. without pressure inside the pipe



## Parameters defining the pressure pulse acting on the wall:

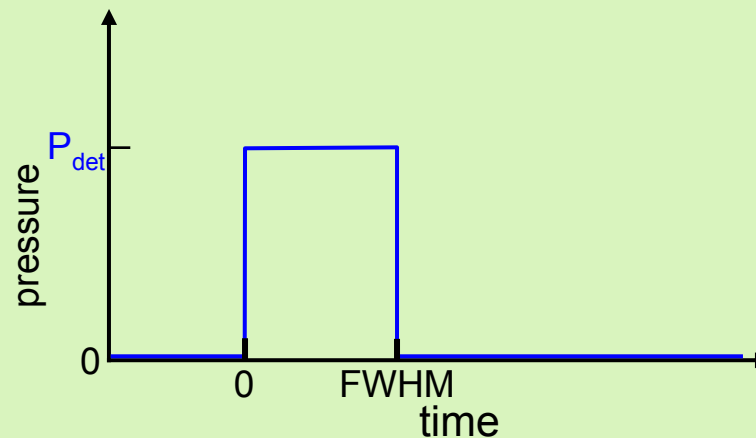
Overall shape of detonation peak is a rectangle;

Pressure is „switched on“ at  $t = 0$ ;

$P_{det}$  = height of detonation peak;

FWHM = full width at half maximum of peak;

Approximation that pressure acts over the entire length of the pipe at the same time.



# Solution for the time dependent displacement $r(t)$ of the wall from the position of rest (fundamental radial oscillation mode)

Time range I:  $t < 0$

$$r(t) = 0$$

Time range II:  $0 \leq t \leq FWHM$

$$r(t) = \frac{K2}{K1} \cdot (1 - \cos(\omega \cdot t))$$

Time range III:  $t > FWHM$

$$r(t) = A \cdot \cos(\omega \cdot t) + B \cdot \sin(\omega \cdot t) \\ = \sqrt{A^2 + B^2} \cdot \sin\left(\omega \cdot t + \arctan\left(\frac{A}{B}\right)\right)$$

$$\omega = 2 \cdot \pi \cdot \nu = \sqrt{K1}$$

$$\nu = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{4 \cdot E}{\rho \cdot (R_o + R_i)^2}} = \frac{1}{2 \cdot \pi \cdot R_{mean}} \cdot \sqrt{\frac{E}{\rho}}$$

$$T = \frac{1}{\nu}$$

$$K1 = \frac{4 \cdot E}{\rho \cdot (R_o + R_i)^2} = \frac{E}{\rho \cdot R_{mean}^2}$$

$$A = \frac{K2}{K1} \cdot (\cos(\omega \cdot FWHM) - 1)$$

$$K2 = \frac{2 \cdot R_i \cdot P_{det}}{\rho \cdot (R_o^2 - R_i^2)} \cong \frac{P_{det}}{\rho \cdot s}$$

$$B = \frac{K2}{K1} \cdot \sin(\omega \cdot FWHM)$$

# Maximum displacements in case that $FWHM \geq T/2$

In time range II  $r(t)$  adopts at least once its maximum value, which is given by:

$$r_{maxII} = 2 \cdot \frac{K2}{K1} = \frac{(R_o + R_i) \cdot R_i}{s \cdot E} \cdot P_{det} \cong \frac{2 \cdot R_{mean}^2}{s \cdot E} \cdot P_{det} \quad (20)$$

The largest displacement in time range III is given by:

$$r_{maxIII} = 2 \cdot \frac{K2}{K1} \cdot \sin\left(\pi \cdot \frac{FWHM}{T}\right) \quad (21)$$

Obviously  $r_{maxIII} \leq r_{maxII}$  and hence the largest displacement is given by the equation 20 for  $r_{maxII}$ .

Note that this is exactly twice the value that would have been obtained if the pressure had not been switched on suddenly but had been increased slowly from zero to  $P_{det}$  ( $\sigma = P \cdot \phi_i / (2 \cdot s) = P \cdot 2 \cdot R_{mean} / (2 \cdot s) = P \cdot R_{mean} / s$  ;  
 $r(t) / R_{mean} = \varepsilon = \sigma / E \Rightarrow r_{max} = P_{det} \cdot R_{mean}^2 / (s \cdot E)$  ).

## Maximum displacements in case that $FWHM < T/2$

In time range II  $r(t)$  increases monotonously with  $t$ , but never reaches the value given by eq. (20), because the pressure of the detonation peak which is the driving force for the wall displacement drops down to zero before  $r(t)$  would have attained its maximum displacement at  $t = T/2$  for the first time. However, for a short period after the pressure dropped down to zero the outwardsly directed movement of the wall continues due to the kinetic energy stored in the moving wall at time  $t = FWHM$ . The largest displacement is hence attained in time range III and is given by eq. (21):

$$r_{maxIII} = 2 \cdot \frac{K2}{K1} \cdot \sin\left(\pi \cdot \frac{FWHM}{T}\right) \quad (21)$$

Obviously, the largest displacement in case that  $FWHM < T/2$  is less than the largest displacement reached in case that  $FWHM \geq T/2$ .

# Introduction of damping factor D in case that FWHM < T/2

## Definition of D:

D is the ratio between the largest displacement in case that FWHM < T/2 („long“ detonative pressure pulse) and the largest displacement in case that FWHM > T/2 („short“ detonative pressure pulse).

Hence D quantifies the damping of the displacement of the wall due to the inertia of the wall material.

$$D = \sin\left(\pi \cdot \frac{FWHM}{T}\right)$$

## Convenient procedure to simplify the „handling“ of D:

- Express FWHM as fraction of T, e. g. FWHM = T/α, α > 2
- Use approximation that sin(x) = x for small x

$$D = \frac{\pi}{\alpha}$$

## Example:

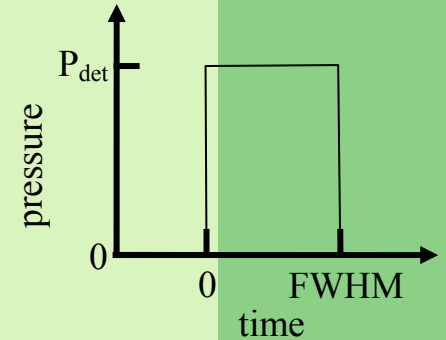
If the width of the detonation peak is one twentieth of T (i.e. α = 20), one finds D = 0.157. Hence the maximum radial displacement and the corresponding circumferential stress only amount to 15.7% of the values both parameters would have attained if the width of the detonation peak had been longer than T/2.

# Example 1a: pipe 1080 x 130, $P_{det} = 325$ bar, FWHM = 1900 $\mu$ s

## Parameters of the exciting detonative pressure pulse

detonative pressure pulse is approximated by a rectangular-shaped pressure-time trace (see sketch to the right)

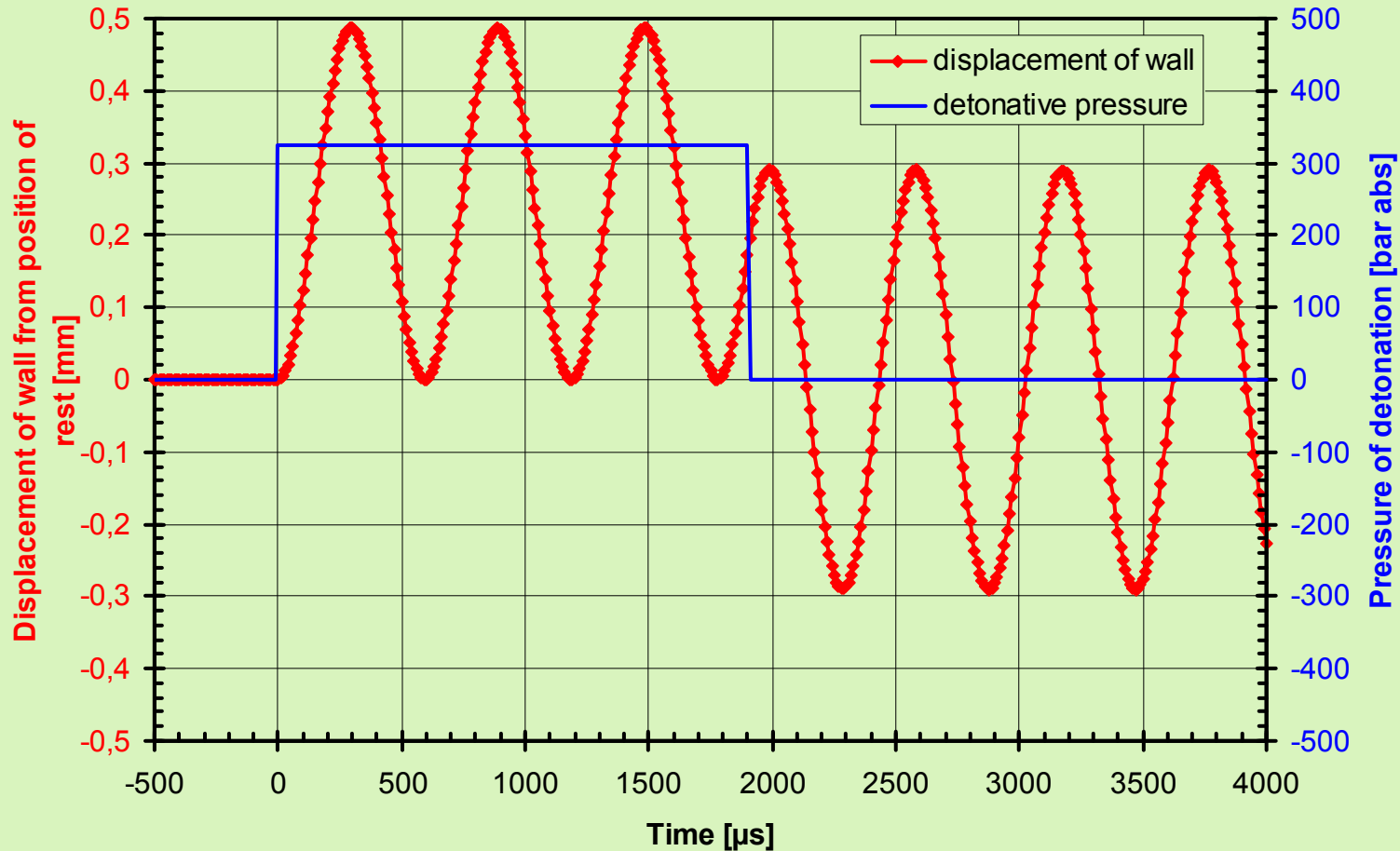
height of the detonative pressure pulse  $P_{det}$  325 bar  
width of the detonative pressure pulse FWHM 1900  $\mu$ s



## Parameters of the pipe exposed to the detonative pressure pulse

outer diameter of pipe $\phi_o$	1080 mm	
inner diameter of pipe $\phi_i$	820 mm	
wall thickness $s$	130 mm	
outer radius of pipe $R_o$	540 mm	
inner radius of pipe $R_i$	410 mm	
mean radius of pipe $R_{mean}$	475 mm	
density $\rho$ of wall material	7900 kg/m <sup>3</sup>	
E-modulus of wall material $E$	200000 N/mm <sup>2</sup>	(typical for all steels)
mass per length of pipe $M_l$	3065,095 kg/m	
inner volume per length	528,102 liter/m	
frequency $\nu$ of fundamental radial oscillation mode	1685,885 Hz	
cycle Time $T$ of fundamental radial oscillation mode	0,59316023 milliseconds	
angular frequency $\omega$	10592,7285 radian/s	
damping $D$	1	

# Example 1a: pipe 1080 x 130, $P_{det} = 325 \text{ bar}$ , $FWHM = 1900 \mu\text{s}$

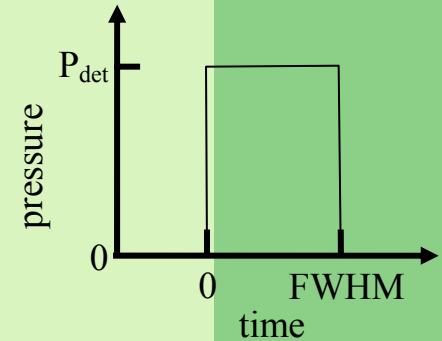


# Example 1b: pipe 1080 x130, $P_{det} = 325$ bar, FWHM=94 $\mu$ s

## Parameters of the exciting detonative pressure pulse

detonative pressure pulse is approximated by a rectangular-shaped pressure-time trace (see sketch to the right)

height of the detonative pressure pulse  $P_{det}$  325 bar  
width of the detonative pressure pulse FWHM 94  $\mu$ s

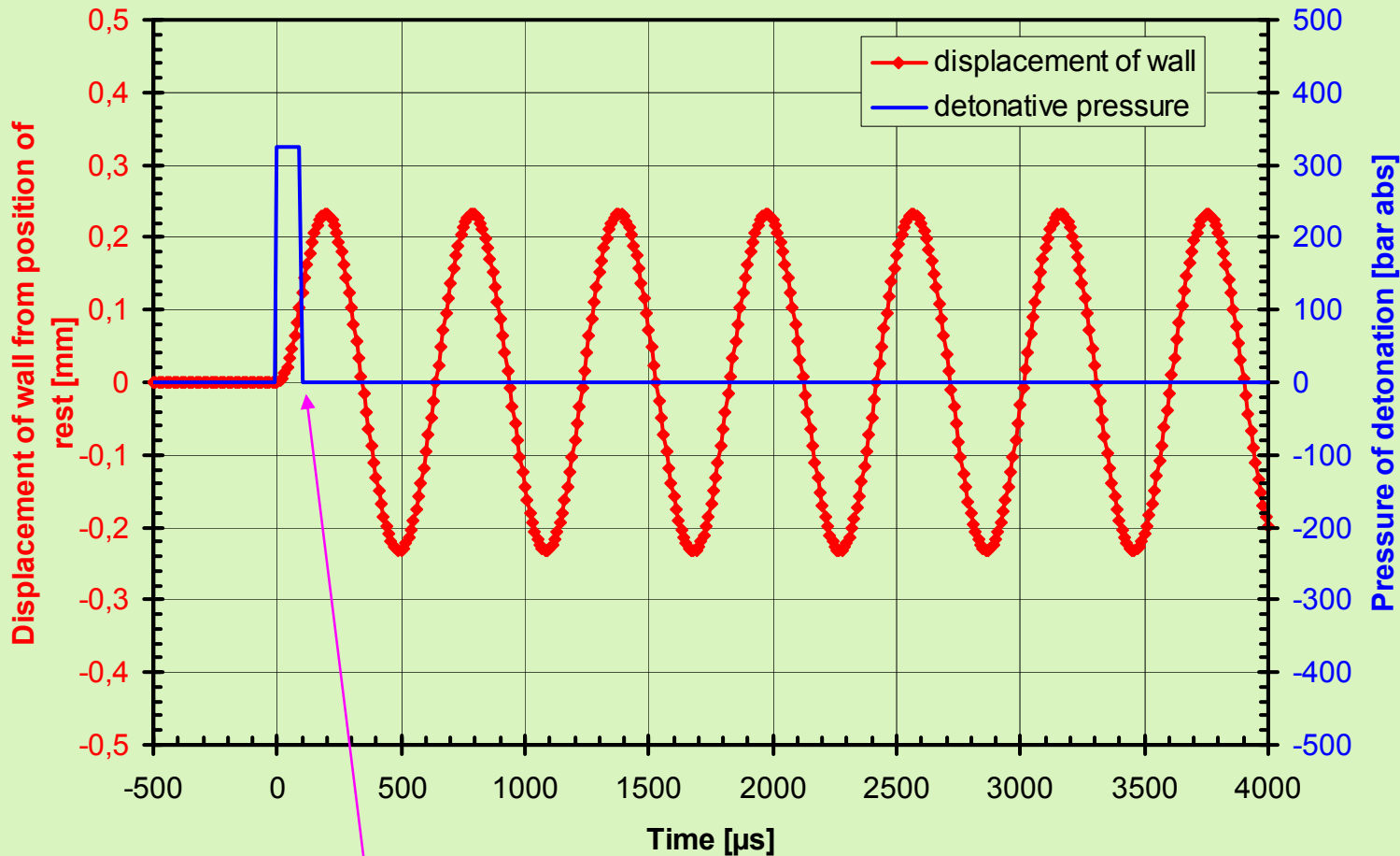


## Parameters of the pipe exposed to the detonative pressure pulse

outer diameter of pipe $\phi_o$	1080 mm	
inner diameter of pipe $\phi_i$	820 mm	
wall thickness $s$	130 mm	
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mass per length of pipe $M_l$	3065,095 kg/m	
inner volume per length	528,102 liter/m	
frequency $\nu$ of fundamental radial oscillation mode	1685,885 Hz	
cycle Time $T$ of fundamental radial oscillation mode	0,59316023 milliseconds	
angular frequency $\omega$	10592,7285 radian/s	
damping $D$	0,47754487	



# Example 1b: pipe 1080 x130, $P_{det} = 325$ bar, FWHM=94 $\mu$ s



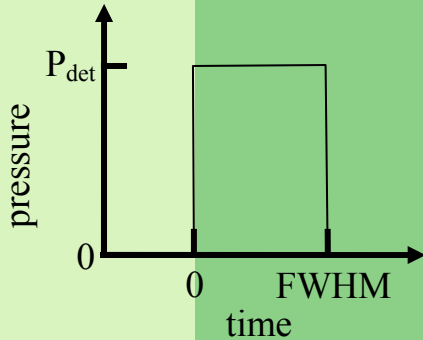
Width of detonative pressure FWHM is less than  $T/2$   
=> Pressure drops to zero before wall has undergone half an oscillation cycle  
=> Maximum displacement is less than in case that  $FWHM \geq T/2$

# Example 2a: pipe 114.3 x 6.02, $P_{det} = 200$ bar, FWHM = 100 $\mu$ s

## Parameters of the exciting detonative pressure pulse

detonative pressure pulse is approximated by a rectangular-shaped pressure-time trace (see sketch to the right)

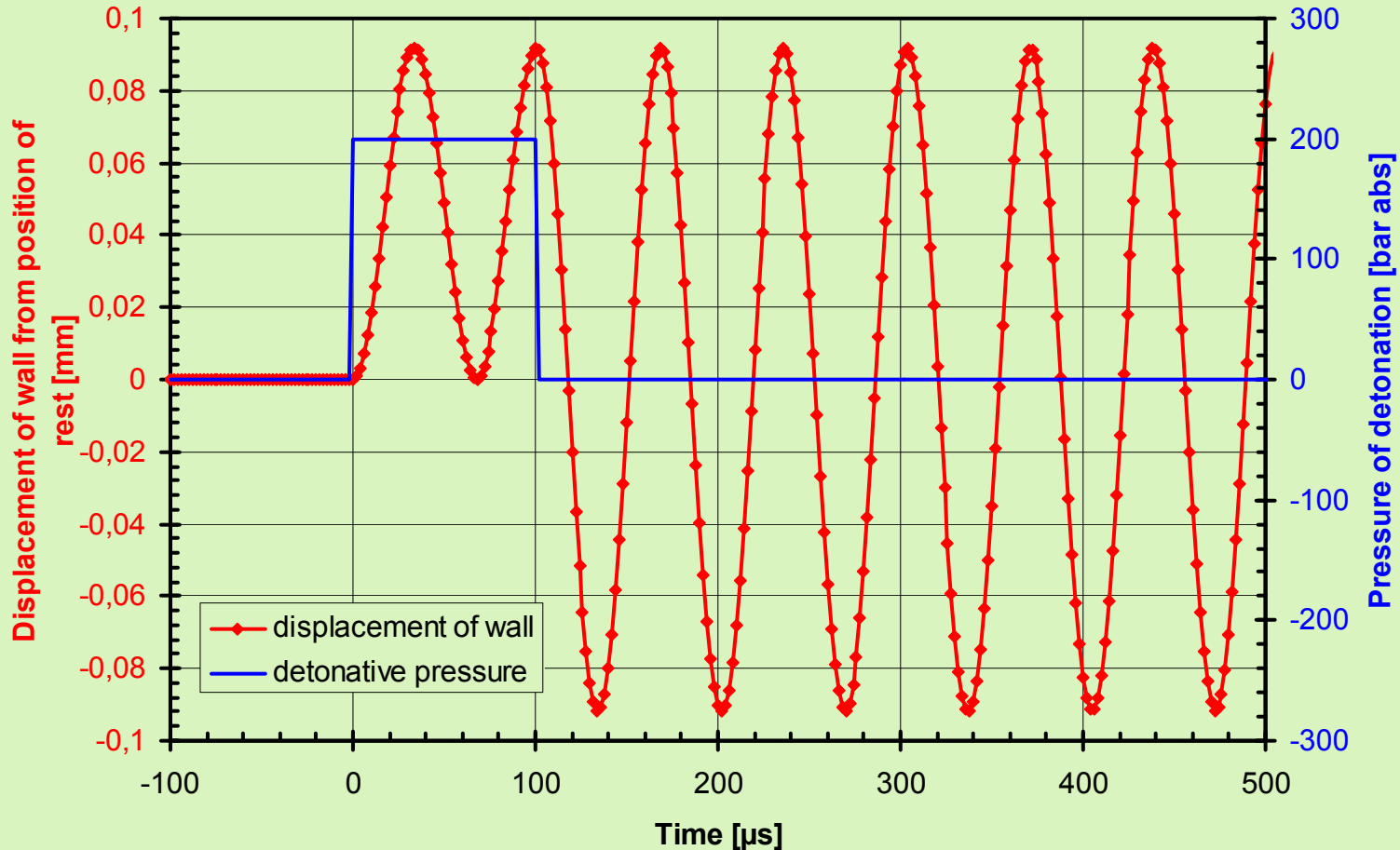
height of the detonative pressure pulse $P_{det}$	200 bar
width of the detonative pressure pulse FWHM	100 $\mu$ s



## Parameters of the pipe exposed to the detonative pressure pulse

outer diameter of pipe $\phi_o$	114,3 mm	
inner diameter of pipe $\phi_i$	102,26 mm	
wall thickness $s$	6,02 mm	
outer radius of pipe $R_o$	57,15 mm	
inner radius of pipe $R_i$	51,13 mm	
mean radius of pipe $R_{mean}$	54,14 mm	
density $\rho$ of wall material	7900 kg/m <sup>3</sup>	
E-modulus of wall material $E$	200000 N/mm <sup>2</sup>	(typical for all steels)
mass per length of pipe $M_l$	16,178 kg/m	
inner volume per length	8,213 liter/m	
frequency $\nu$ of fundamental radial oscillation mode	14791,197 Hz	
cycle Time $T$ of fundamental radial oscillation mode	0,06760778 milliseconds	
angular frequency $\omega$	92935,834 radian/s	
damping $D$	1	

# Example 2a: pipe 114.3 x 6.02, $P_{det} = 200$ bar, FWHM = 100 $\mu$ s

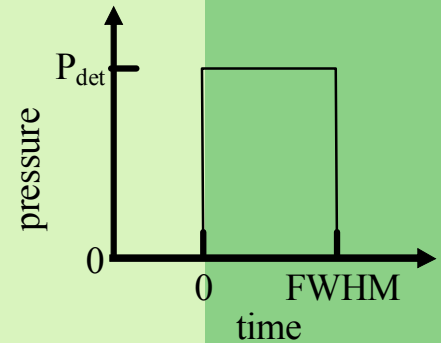


# Example 2b: pipe 114.3 x 6.02, $P_{det} = 200$ bar, FWHM = 10 $\mu$ s

## Parameters of the exciting detonative pressure pulse

detonative pressure pulse is approximated by a rectangular-shaped pressure-time trace (see sketch to the right)

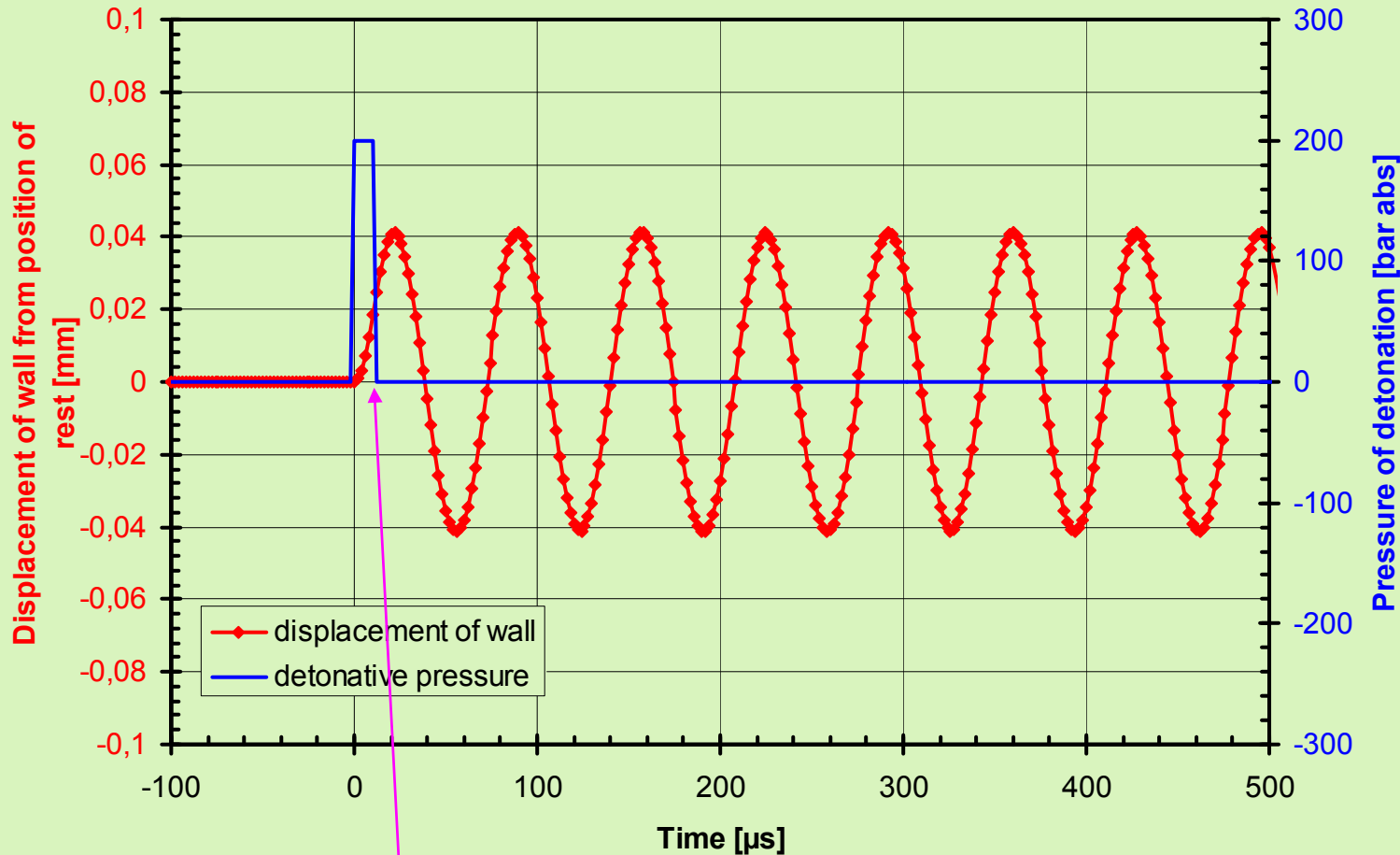
height of the detonative pressure pulse $P_{det}$	200 bar
width of the detonative pressure pulse FWHM	10 $\mu$ s



## Parameters of the pipe exposed to the detonative pressure pulse

outer diameter of pipe $\rightarrow o$	114,3 mm	
inner diameter of pipe $\rightarrow i$	102,26 mm	
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frequency $\rightarrow$ of fundamental radial oscillation mode	14791,197 Hz	
cycle Time $T$ of fundamental radial oscillation mode	0,06760778 milliseconds	
angular frequency $\rightarrow$	92935,834 radian/s	
damping $D$	0,44813601	

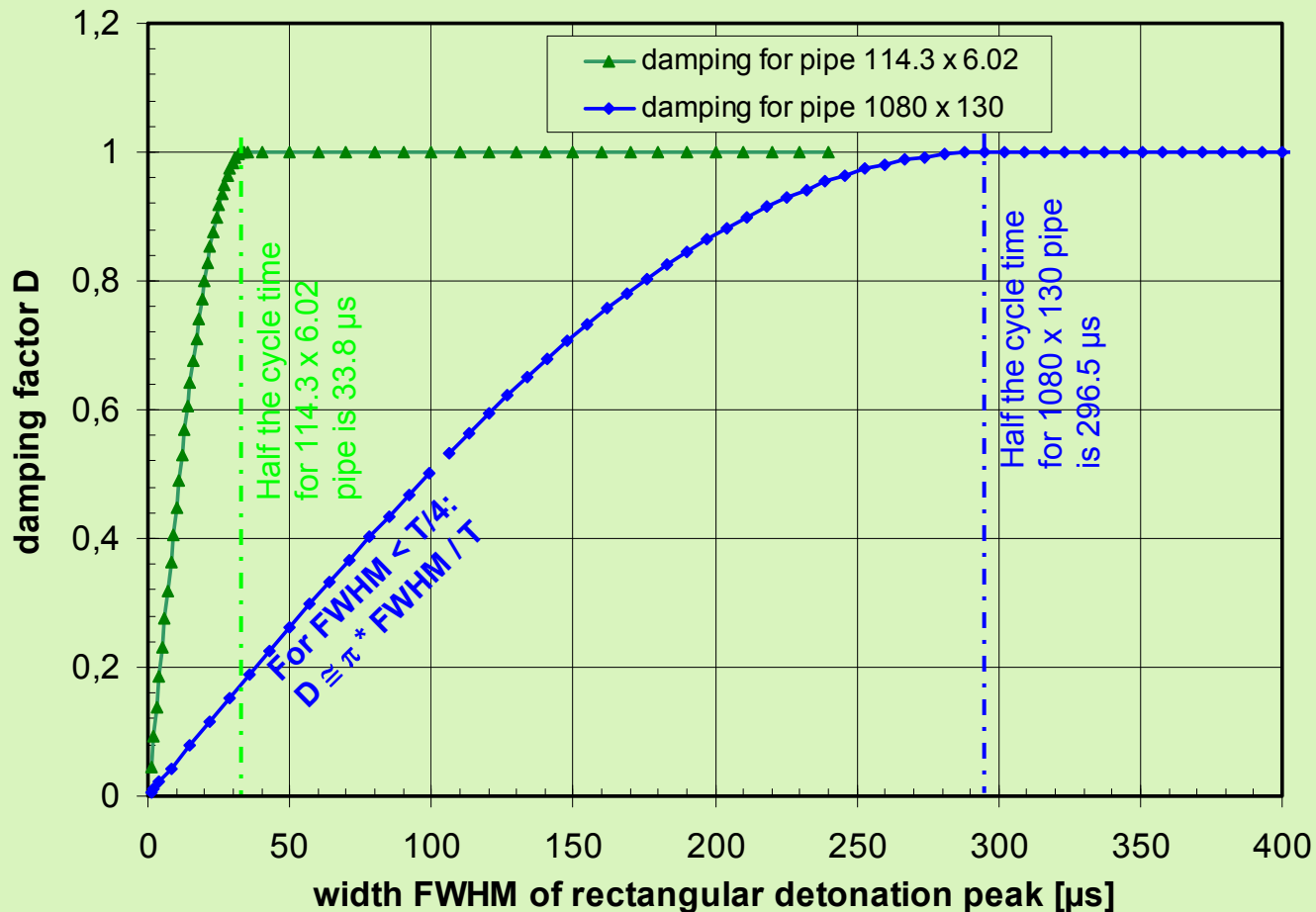
# Example 2b: pipe 114.3 x 6.02, $P_{det} = 200$ bar, FWHM = 10 $\mu$ s



Width of detonative pressure FWHM is less than  $T/2$   
=> Pressure drops to zero **before** wall has undergone half an oscillation cycle  
=> Maximum displacement is less than in case that  $FWHM \geq T/2$

# Damping of the displacement of the wall in case of detonation peaks short compared to the cycle time of the fundamental radial oscillation mode

$$\text{Damping factor } D = \frac{\text{Maximum displacement of wall obtained for actual value of FWHM}}{\text{Maximum displacement of wall obtained for FWHM} > T/2}$$



# What is all missing in this simple model

- Actually the detonative pressure pulse is not present over the entire length of the pipe at the same time, but travels through. For certain propagation speeds there will be some nasty positive feedback effects between the detonation and flexural waves in the pipe wall.
- Higher order breathing modes not considered. (Will they be excited at all?)
- Bending modes not considered. (Will they be excited at all?)
- Potential mode coupling effects not considered.  
(Breathing modes of cylindrical pipe are orthogonal to each other, coupling can not occur. But how about bending and breathing modes?)
- Effect of flange connections on oscillation behaviour not considered.
- Deviations from perfect cylindrical geometry not considered.

Ovality can be neglected. Deviations of wall thickness over circumference can be neglected as far as this is due to the manufacturing process, but corrosive attack might reduce wall thickness at bottom of horizontally mounted pipes by up to 15%). Welding seams can be neglected

References for real pipe geometries:

DIN EN ISO 1127 „Stainless steel tubes, Dimensions, Tolerances and conventional masses per unit length

DIN EN 10217 “Welded steel tubes for pressure purposes – Technical delivery conditions”, parts 1, 2, 3, 5, 7

Public Available Specification PAS 1057 „Pipe classes for process plants”

- Introduction
- For which  $p_{\text{det}}$ -scenarios seems the corresponding mechanical load  $p_{\text{stat}}$  to be clear
- For which  $p_{\text{det}}$ -scenarios is the corresponding mechanical load  $p_{\text{stat}}$  not yet clear
- What do experiments suggest
- Why are systematic experiments difficult
- Simple analytic approach („breathing mode“) to quantify radial displacement of wall in dependence of duration of detonative pressure pulse
- What is our hope concerning the mechanical load  $p_{\text{stat}}$  brought about by detonative pressure pulses
- What might help to answer the open questions with regard to mechanical load  $p_{\text{stat}}$



# What is our hope?

## **Hope 1:**

At the location where the DDT occurs and also closely downstream of it the duration of the detonation peak should be extremely short.

## **If this were true:**

Although the height of the peak may be a factor of 4 to 8 larger than the height of the peak of the stable CJ-Detonation, the damping factor  $D$  will be much less than 1 and thus  $p_{\text{stat}}$  is only a factor 1.5 to 2 larger than in those regions where the stable CJ detonation propagates.

## **Hope 2:**

Neither higher order modes nor mode coupling play a relevant role.

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- What might help to answer the open questions with regard to mechanical load  $p_{\text{stat}}$

# The (hopefully) ultimate solution

- Calculate **reliable** pressure profiles  $p_{\text{det}}(x)^*$  at the moment of DDT and in the short time intervals afterwards when the detonation still propagates through the region where the mixture got compressed by its inertia and by frictional forces with the wall.

If possible the calculation should also include the various degrees of precompression and the reflection of the detonative peak.

- Use these pressure profiles as a source term in FE calculations to quantify the response of the containment (pipes and vessels) subjected to these events.

These calculations should include:

- a) interference effects of the different vibration modes of the pipe
- b) potential mode coupling effects
- c) decay of vibration modes due to energy dissipation
- d) critical propagation speeds of the detonation (interaction between propagating detonation and flexural waves in the wall)
- e) effects of elbows, flange connections, blind flanges
- f) realistic deviations from cylindrical symmetry of the wall material

\*: let  $x$  denote the axial position in the pipe